## PART B – MATHEMATICS

**31.** **Statement–1 :** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + .... + (361 + 380 + 400)$ is 8000.

**Statement–2 :** $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

**Ans.** (3)

**32.** An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

(1) $x^2 + 4y^2 = 16$
(2) $4x^2 + y^2 = 4$
(3) $x^2 + 4y^2 = 8$
(4) $4x^2 + y^2 = 8$

**Ans.** (1)

**33.** The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(1) 5/3 (2) 10/3
(3) 3/5 (4) 6/5

**Ans.** (2)

**34.** Let P and Q be $3 \times 3$ matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

(1) –1 (2) –2
(3) 1 (4) 0

**Ans.** (4)

**35.** If n is a positive integer, then

$\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is :

(1) a rational number other than positive integers
(2) an irrational number
(3) an odd positive integer
(4) an even positive integer

**Ans.** (2)

**36.** **Statement–1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

**Statement–2 :** If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

**Ans.** (3)

**37.** Three numbers are chosen at random without replacement from {1, 2, 3, ..... , 8}. The probability that their minimum is 3, given that their maximum is 6, is :

(1) $\frac{2}{5}$ (2) $\frac{3}{8}$
(3) $\frac{1}{5}$ (4) $\frac{1}{4}$

**Ans.** (3)

**38.** If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals :

(1) $g(x) \cdot g(\pi)$ (2) $\frac{g(x)}{g(\pi)}$
(3) $g(x) + g(\pi)$ (4) $g(x) - g(\pi)$

**Ans.** (3 & 4)

**39.** Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :-

(1) 879 (2) 880 (3) 629 (4) 630

**Ans.** (1)

**40.** If 100 times the $100^{th}$ term of an A.P. with non-zero common difference equals the 50 times its $50^{th}$ term, then the $150^{th}$ term of this A.P. is :

(1) zero (2) –150
(3) 150 times its $50^{th}$ term (4) 150

**Ans.** (1)

**41.** The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is :

(1) $10\sqrt{2}$ (2) $20\sqrt{2}$ (3) $\frac{10\sqrt{2}}{3}$ (4) $\frac{20\sqrt{2}}{3}$

**Ans.** (4)

**42.** An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

(1) $x - 2y + 2z + 5 = 0$
(2) $x - 2y + 2z - 3 = 0$
(3) $x - 2y + 2z + 1 = 0$
(4) $x - 2y + 2z - 1 = 0$

**Ans.** (2)

**43.** The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

(1) exactly four real roots.
(2) infinite number of real roots.
(3) no real roots.
(4) exactly one real root.

**Ans.** (3)

**44.** The negation of the statement "If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school.
(2) I will become a teacher and I will not open a school.
(3) Either I will not become a teacher or I will not open a school.
(4) Neither I will become a teacher nor I will open a school.

**Ans.** (2)

**45.** The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\ p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is:

(1) ln18 (2) 2 ln18 (3) ln9 (4) $\frac{1}{2}$ ln18

**Ans.** (2)

**46.** If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a \ln|\sin x - 2\cos x| + k$ then a is equal to :

(1) 2 (2) –1 (3) –2 (4) 1

**Ans.** (1)

**47.** Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

**Ans.** (4)

**48.** A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) –4 (4) –2

**Ans.** (4)

**49.** Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is :

(1) $5^3$ (2) $5^2$ (3) $3^5$ (4) $2^5$

**Ans.** (3)

**50.** Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by :

(1) $\vec{r} = -3\vec{q} + \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$

(2) $\vec{r} = 3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right)\vec{p}$

(4) $\vec{r} = \vec{q} - \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right)\vec{p}$

**Ans.** (3)

**51.** If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :

(1) $\frac{11}{5}$ (2) $\frac{29}{5}$ (3) 5 (4) 6

**Ans.** (4)

**52.** In a $\Delta PQR$, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

**Ans.** (3)

**53.** Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If $u_1$ and $u_2$ are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

(1) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

**Ans.** (1)

**54.** If $f : R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is :

(1) continuous only at x = 0.
(2) continuous for every real x.
(3) discontinuous only at x = 0.
(4) discontinuous only at non-zero integral values of x.

**Ans.** (2)

**55.** A spherical balloon is filled with $4500\pi$ cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of $72\pi$ cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :

(1) 9/2 (2) 9/7 (3) 7/9 (4) 2/9

**Ans.** (4)

**56.** Let $a, b \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

**Statement–1 :** f has local maximum at $x = -1$ and at $x = 2$.

**Statement–2 :** $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

**Ans.** (3)

**57.** If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

(1) on the imaginary axis.
(2) either on the real axis or on a circle passing through the origin.
(3) on a circle with centre at the origin.
(4) either on the real axis or on a circle not passing through the origin.

**Ans.** (2)

**58.** Consider the function,
$f(x) = |x - 2| + |x - 5|$, $x \in R$.
**Statement–1 :** $f'(4) = 0$.
**Statement–2 :** f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5).

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

**Ans.** (4)

**59.** Let $x_1, x_2, \ldots, x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and $\sigma^2$ be their variance.
**Statement–1 :** Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.
**Statement–2 :** Arithmetic mean of $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

**Ans.** (1)

**60.** If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to:

(1) 0 (2) – 1 (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

**Ans.** (4)